See-saw Mechanism and Possible Generation Structure

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Abstract

On the basis of see-saw mechanism, possible generation structure is investigated. Assuming that right-handed neutrinos are realistic particles below GUTs mass, it is shown that there is a choice of new scheme of generation, where the combination of leptons and quarks is different from the ordinary one. Our scheme predict a new proton decay mode.

§1. Introduction

The problem of generation structure which is observed in low energy region seems to exhibit an important suggestion to disclose fundamental existence-form of matter.[1),2)] In investigating to that structure, neutrino problem may be situated at the capstone. It seems that the see-saw mechanism is the most prominent one to explain the smallness of neutrino mass, where its smallness is reduced to largeness of the right-handed Mayorana neutrino mass which is often assumed to be the order of GUTs mass. In the GUTs scale, a new physics beyond to our present prediction may appear, and it is not clear whether the right-handed Mayorana neutrino with GUTs mass can be treated as ordinary particle. On the other hand, if we assume that the right-handed Mayorana neutrino possesses mass below GUTs scale, and can be completely treated as ordinary particles described by ordinary gauge field theory, possible alternative generation structure will be implied.

In this paper, we will investigate possible new generation structure on the basis of viewpoint suggested by the see-saw mechanism, and discuss on a few typical new features due to this scheme.

§2. See-saw mechanism of neutrino mass splitting

For definiteness, we will make a quick review of see-saw mechanism of neutrino mass generation.[3)] As a basis of construction of our scheme, let us consider the D-M (Dirac-Mayorana) mass term[4)-6)] in the simplest case of one generation labeled by the generation suffix i. We have

$$\begin{aligned} \mathcal{L}^{D-M} &= -\frac{1}{2}m_{iL}\overline{(\nu_{iL})^c}\nu_{iL} - m_{iD}\bar{\nu}_{iR}\nu_{iL} - \frac{1}{2}m_{iR}\bar{\nu}_{iR}(\nu_{iR})^c \quad + \quad h.c. \\ &= -\frac{1}{2}\overline{\begin{pmatrix} (\nu_{iL})^c \\ \nu_{iR} \end{pmatrix}}\; M \; \begin{pmatrix} \nu_{iL} \\ \nu_{iR}^c \end{pmatrix} \quad + \quad h.c. \end{aligned} \tag{2·1}$$

Here

$$M = \begin{pmatrix} m_{iL} & m_{iD} \\ m_{iD} & m_{iR} \end{pmatrix}, \tag{2·2}$$

m_{iL}, m_{iD}, m_{iR} are parameters. For a symmetrical matrix M we have

$$M = U\ m\ U^{\dagger}, \tag{2·3}$$

where $U^{\dagger}U = 1$, $m_{jk} = m_j\delta_{jk}$. From Eq.(2·1) and Eq.(2·3) we have

$$\mathcal{L}^{D-M} = -\frac{1}{2}\sum_{\alpha=1}^{2} m_{i\alpha}\bar{\chi}_{i\alpha}\chi_{i\alpha}, \tag{2·4}$$

where

$$\begin{aligned} \nu_{iL} &= \cos\theta_i\chi_{i1L} + \sin\theta_i\chi_{i2L} \\ (\nu_{iR})^c &= -\sin\theta_i\chi_{i1L} + \cos\theta_i\chi_{i2L}. \end{aligned} \tag{2·5}$$

Here χ_{i1} and χ_{i2} are fields of Majorana neutrinos with masses m_{is} $(small), m_{iB}$ (Big), respectively. The masses m_{is}, m_{iB} and the mixing angle θ_i are connected to the parameters m_{iL}, m_{iD} and m_{iR} by the relations

$$\begin{aligned} m_{is} &= \frac{1}{2}\ |m_{iR}\ +\ m_{iL}\ -\ a_i| \\ m_{iB} &= \frac{1}{2}\ |m_{iR}\ +\ m_{iL}\ +\ a_i| \\ \sin 2\theta_i &= \frac{2m_{iD}}{a_i}, \qquad \cos 2\theta_i = \frac{m_{iR} - m_{iL}}{a_i} \end{aligned} \tag{2·6}$$

where

$$a_i = \sqrt{(m_{iR} - m_{iL})^2\ +\ 4m_{iD}^2} \tag{2·7}$$

It should be noted that the relations Eq.(2·6) are exact ones. Let us assume now that

$$m_{iL} = 0,\ m_{iD} \simeq m_{iF},\ m_{iR} \gg m_{iF}, \tag{2·8}$$

where m_{iF} is the typical mass of the leptons and quarks of the generation labeled by suffix i. From Eq.(2·6) we have

$$m_{is} \simeq \frac{m_{iF}^2}{m_{iR}},\quad m_{iB} \simeq m_{iR},\quad \theta_i \simeq \frac{m_{iD}}{m_{iR}} \tag{2·9}$$

Thus, if the conditions Eq.(2·8) are satisfied, the particles with definite masses are split to a very light Majorana neutrino with mass $m_{is} \ll m_{iF}$ and a very heavy Majorana particle with mass $m_{iB} \simeq m_{iR}$. The current neutrino field ν_{iL} practically coincides with χ_{i1L} and $\chi_{i2} \simeq \nu_{iR}\ +\ (\nu_{iR})^c$, because θ_i is extremely small.

That is , we have assumed such scheme that in D-M mass term Dirac masses are of order of usual fermion masses, the right-handed Majorana masses, responsible for lepton numbers violation, are extremely large and the left-handed Majorana masses are equal zero. In such a

scheme neutrinos are Majorana particles with masses much smaller than masses of the other fermions. The predictions of neutrino masses depend on the value of the m_{iR} mass. The value of m_{iR} is often assumed that $m_{iR} = M_{GUT}$, where M_{GUT} is grand unification scale. Though this value depends on the model, a typical one is $m_R \simeq 10^{19}$ GeV (Planck mass). In the M_{GUT} region, the ordinary particle picture may be drastically changed.

It should be emphasized that, however, there is no definite reason why the mass of m_{iR} should be M_{GUT}. It is also possible that though the mass of m_{iR} is very huge it is below the Planch mass and possessing the picture of ordinary particle. In such case, possibility of new generation structure will be implied, which is discussed in the next section.

§3. Possible generation structure based on see-saw mechanism

The "standard" structure of generation is composed of each leptons and quarks, which generation number is labeled according to the sequence of their historical discovery, depending on the energy frontier. It should be emphasized that, in the present stage, the only compelling reason to relate leptons and quarks is the anomaly free condition, and no essential principle to compose the generation is ever unknown. [7] Then, regarding the conventional rule to classify the generation according to the sequence of magnitude of the total masses belonging to the same suffix as the fundamental rule which should be founded on the deeper level, we will investigate alternative possible generation structure.

If the right-handed Majorana neutrinos are realistic particles below the GUT mass and responsible to the see-saw mechanism, its existence will affect to the above mentioned classification procedure of the generations. From Eq.(2·9),

$$m_{iR} \simeq \frac{m_{iF}^2}{m_{is}}. \tag{3·1}$$

Remembering to the fact that the current neutrino field ν_{iL} practically coincides with χ_{i1L} with the mass m_{is} together with the above relation Eq.(3·1), let us investigate the new generation scheme with realistic m_{iR}. Though the values of neutrino mass has not yet been established, [8] a prominent possibility is that the mass of neutrinos exhibits an extreme hierarical structure [9],

$$m_{\nu_e} \ll m_{\nu_\mu} \ll m_{\nu_\tau}. \tag{3·2}$$

The degree of the mass difference in Eq.(3·2) will read to a suggestion to find possible new generation structure. That is, if the degree of mass difference of neutrinos is larger than that of the representative mass of each generation m_{iF}, the magnitude of total mass of i-th neutrinos, i.e. $m_{\nu_i} + m_{iR}$, will become in reverse order to label number i, and this magnitude

will dominate the total lepton masses belonging to i-th label. Then, provided that we adopt the custom to nominate the generation number "from light to heavy" particles, we will conclude the following generation classification,

$$\begin{array}{llll} I & (\nu_{1R})^c & \binom{\nu_\tau}{\tau} & \binom{u_L}{d_L} \\ II & (\nu_{2R})^c & \binom{\nu_\mu}{\mu} & \binom{c_L}{s_L} \\ III & (\nu_{3R})^c & \binom{\nu_e}{e} & \binom{t_L}{b_L} \end{array} \tag{3·3}$$

where we have written only the left-handed components. That is, taking into account of the existence of realistic particles ν_{iR}, our proposal for new generation structure is different from the ordinary one. The characteristic feature is that the leptons belonging to the first and the third generations are exchanged. It should be noted that this result is caused by the existence of realistic ν_{iR} and the see-saw mechanism.

§4. "Generational" and "Inter-generational" gauge bosons in GUTs and proton decay problem

The generation structure is the important feature observed in low energy region. We will suppose that this structure still leaves any trace in high-energy region below GUTs scale. This situation leads to the viewpoint to distinguish the gauge bosons appearing in GUTs containing all generations. That is, the gauge bosons common to all generations, and ones which connect particles belonging to different generations. In SU(5) GUT, only the former type of gauge bosons, W, Z, A and X, Y , appear. We will call this type of gauge bosons as "generational gauge bosons" [10]. In the GUTs structure containing all generations, new gauge bosons connecting particles belonging to the different generations appear generally, and we will call them as "inter-generational gauge bosons". [11] Our viewpoint means that the contribution of the "inter-generational gauge boson" should be more suppressed than that of the "generational gauge bosons" in low and intermediate energy region below the GUTs scale.

If we take this viewpoint, our model of new generation structure lead to different feature of proton decay problem in GUTs. Our scheme predict the proton decay mode due to the generational gauge bosons X and Y

$$P \rightarrow \tau^+ \, M_0 \tag{4·1}$$

instead of the well known mode

$$P \rightarrow e^+ \, M_0 \tag{4·2}$$

where M_0 represents $\pi^0, \rho_0, \omega, \eta, \pi^+\pi^- \cdots$. It should be noted that the process in Eq.(4·1) is forbidden by Q value. The inter-generational gauge bosons may cause the process in Eq.(4·2), however, this process will be extremely suppressed in low and intermediate energy region below GUTs scale. The other mode is

$$P \rightarrow \overline{\nu}_\tau \, M^+ \tag{4·3}$$

with $\pi^+, \rho^+, \pi^+\pi^0 \cdots$, and this process is allowed in our scheme.

Thus, the proton decay in our model is different from well known model of generation structure. That is, the well-known difficulty so-called "proton decay problem" is consistent to our scheme. It should be emphasized that when the proton decay is actually observed it will be made clear whether our scheme is realized or not.

§5. Discussion

In this paper, we have proposed a possible new structure of generation combination of leptons and quarks on the basis of see-saw mechanism and the actual existence of realistic right-handed Mayorana neutrino. We have emphasized classification of the gauge bosons appearing in GUTs, that is the "generational gauge bosons" and the "inter-generational gauge bosons". Our model gives some important predictions to so-called "proton decay problem". In order to give further predictions, some specific assumptions and introduction to many parameters is inevitable in the present stage, then in this paper we have restricted ourselves to proposal of framework of the model.

We have discussed on only one generation case in order to investigate the fundamental framework of new possibility. However, massive neutrinos will generally cause the generation mixing.[Appendix] In fact, it is pointed out that the large mixing will be realized in the atomosphear neutrino events. [12] In the framework of ordinary generation structure, it seems that new mysterious feature may appear provided that we take into account of this large mixing in GUTs mass relation. That is , the generation labels of leptons seem to be exchanged in appearance between the first and the third generation. [13] It should be emphasize that, in our model this appearance is not the exchange but the fundamental generation structure

itself. Further analysis concerning to the problem of generation mixing will be discussed in elsewhere.

Finally, it should be noted that our discussion to introduce the generation combination may be heuristic one based on the historical or conventional rule of generation nomination in the present stage, then our model will maintain its meaning even if some premises are partially altered. This simple rule will be based on the deeper level structure of nature, and will be justified in near future.

Appendix

In the general case of 3-generations, the mixing has, instead of Eq.(2·1),the following form,

$$\begin{aligned}\mathcal{L}_{D-M} = &- \frac{1}{2}\sum_{l,l'} \overline{(\nu_{l'L})^c} M^L_{l'l}\nu_{lL} \ - \sum_{l',l} \bar{\nu}_{l'R} M^D_{l'l}\nu_{lL} \\ &- \frac{1}{2}\sum_{l',l} \bar{\nu}_{l'R} M^R_{l'l}(\nu_{l'R})^c \quad + \quad h.c.,\end{aligned} \tag{A·1}$$

where M^L, M^D and M^R are 3×3 complex matrixes.

It is clear that, neutrinos with definite masses are Majorana particles. As in the case of one generation given in Eq.(2·4), the number of massive particles in this case is twice of the number of lepton flavours.From Eq.(A·1), after standard procedure of the diagonalization of a 6 ×6 matrix M we have

$$\mathcal{L}_{D-M} = -\frac{1}{2}\sum_{\alpha=1}^{6} m_\alpha \bar{\chi}_\alpha \chi_\alpha, \tag{A·2}$$

where $\chi_\alpha = \chi_\alpha^c$ is the field of Majorana neutrinos with mass m_α. The current fields ν_{lL} and fields $(\nu_{lR})^c = C\bar{\nu}^T_{lR}$ (left-handed components) are connected with left-handed components of massive Majorana fields χ_{iL} by a unitary transformation

$$\begin{aligned}\nu_{lL} &= \sum_{\alpha=1}^{6} U_{l\alpha}\chi_{\alpha L} \\ (\nu_{lR})^c &= \sum_{\alpha=1}^{6} U_{\bar{l}\alpha}\chi_{\alpha L}\end{aligned} \tag{A·3}$$

where U is a unitary 6 × 6 matrix.

If all masses m_α are small enough, the fields ν_{lL} are usual flavor left-handed neutrinos and right-handed antineutrinos, while the fields ν_{lR} are "sterile" right-handed neutrinos and left-handed antineutrinos, and do not take place in the standard weak interaction.

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[7] The mass relation in GUTs means merely indirect condition. If new structure of generation is established, the modified GUTs relation will be constructed on the basis of it.

[8] In order to determine the neutrino masses, the mixing structure should be completely clarified. In the present stage, it will be premature to find the definite masses of neutrinos.

[9] On the other hand, the other possibility that the masses of neutrinos are almost the same order may remain in the present stage.
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